

Linear Algebra

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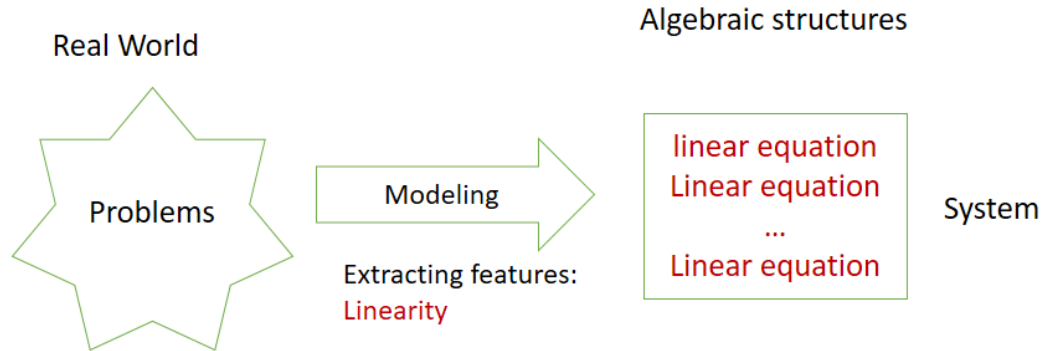
What is linear algebra?

- The study of algebraic structures, including vectors, matrices, and systems of linear equations, used to study properties of linear spaces.
 - Vectors are used to represent positions in space or directions and magnitudes.
 - Systems of linear equations are represented by matrices.
 - Algebraic operations on matrices are used to solve these equations.

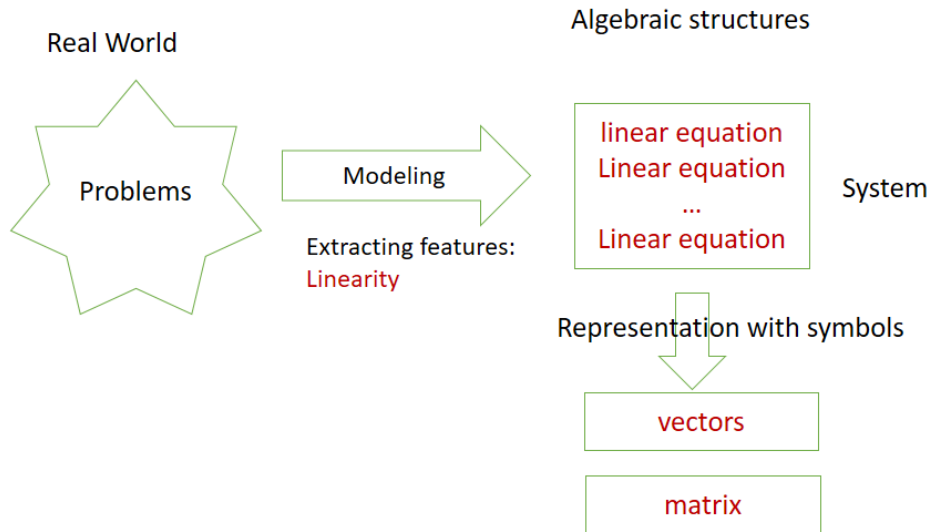
What is linear algebra?(cont.)

- It provides a framework for solving problems that arise in physics, engineering, economics, statistics, and other fields.
- Linear algebra is a fundamental tool in a wide range of fields, including computer graphics, machine learning, data analysis, statistics, physics, and many others. It is a key area for mathematical modeling and problem-solving in these fields.

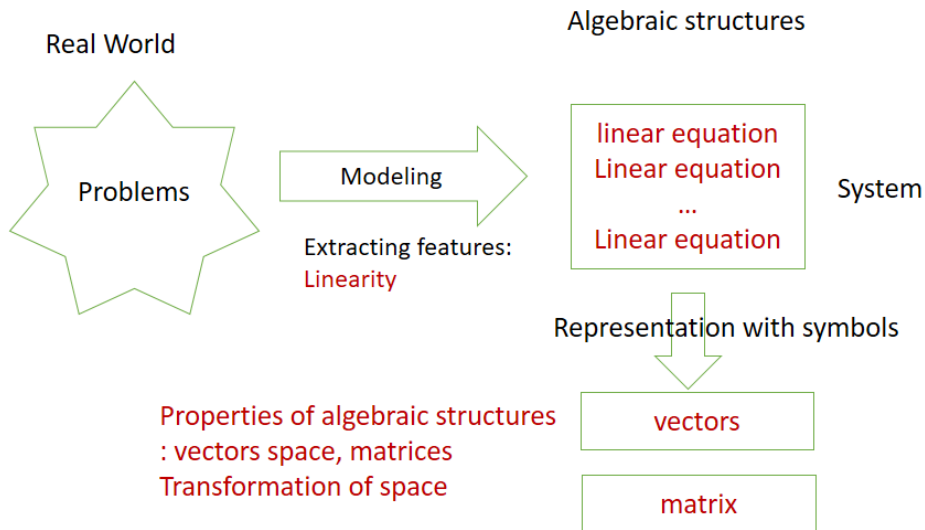
What is linear algebra?(cont.)



What is linear algebra?(cont.)



What is linear algebra?(cont.)



System

input

system

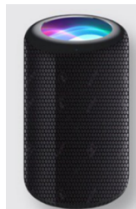
output

Speech Recognition System



AI assistant System

“안녕”



“안녕”

“좋은 아침이네요”

Translation System

“좋은 아침이네요”

“Good Morning”

A system has input and output. (function, transformation, operator)

Linearity of linear system

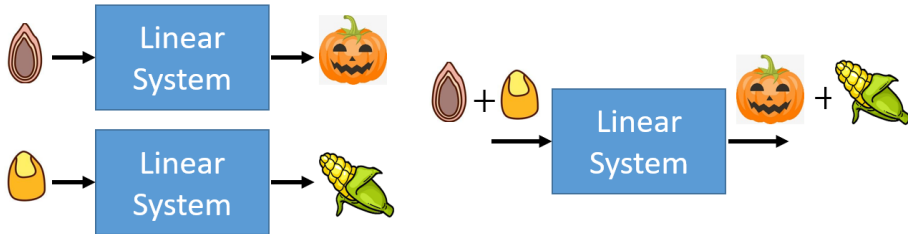
- Linearity is a property that describes a system's response to an input signal. Linearity refers to the property that the system T satisfies two conditions:
 - 1 Additivity(addition): for any inputs u , u' , v and v' , a system T satisfies the following addition condition: u and v produce output signals $T(u) = u'$ and $T(v) = v'$ respectively, then the input signal $u + v$ will produce an output signal $T(u + v) = (u + v)' = u' + v' = T(u) + T(v)$.
 - 2 Homogeneity(multiplication): for any vectors u and any scalars λ , a system has input λu and produces an output signal, then scaling the input signal by a constant factor λ produces a corresponding output signal $T(\lambda u) = \lambda T(u)$.
- Linearity is a fundamental property of many physical, mathematical, and engineering systems. This property allows us to use powerful tools and techniques from linear algebra to analyze and understand the behavior of the system.

Linear system

- 1. Homogeneity(multiplication)



- 2. Additivity(addition)



Linear system(cont.)

- 1. Homogeneity(multiplication)

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \text{Linear System} \rightarrow \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$\begin{matrix} k \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ k = 2 \\ \begin{bmatrix} 2 \\ 4 \end{bmatrix} \end{matrix} \rightarrow \text{Linear System} \rightarrow \begin{matrix} k \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \\ \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix} \end{matrix}$$

- 2. Additivity(addition)

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \text{Linear System} \rightarrow \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

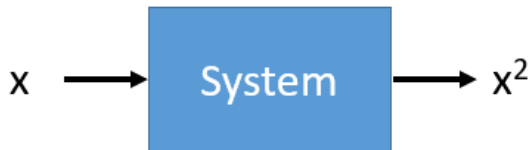
$$\begin{bmatrix} 5 \\ 6 \end{bmatrix} \rightarrow \text{Linear System} \rightarrow \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

$$\begin{matrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} \\ \begin{bmatrix} 6 \\ 8 \end{bmatrix} \end{matrix} \rightarrow \text{Linear System} \rightarrow \begin{matrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \\ \begin{bmatrix} 10 \\ 12 \\ 14 \end{bmatrix} \end{matrix}$$

Linearity Check

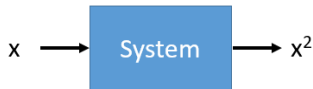
Linear?

$$f(x) = x^2$$

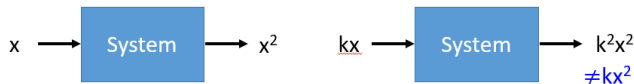


Linearity Check(cont.)

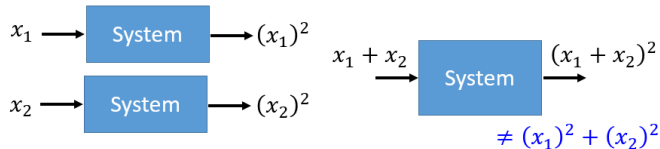
Linear : **NO**



- 1. Homogeneity(multiplication)



- 2. Additivity(addition)



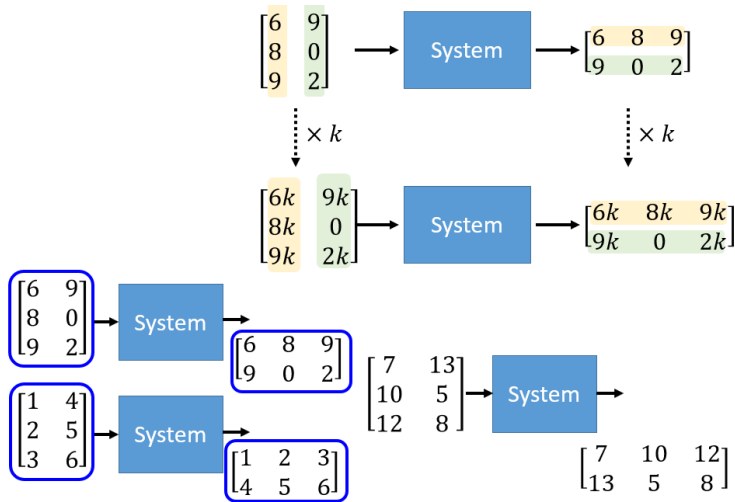
Linearity Check(cont.)

Linear?



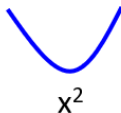
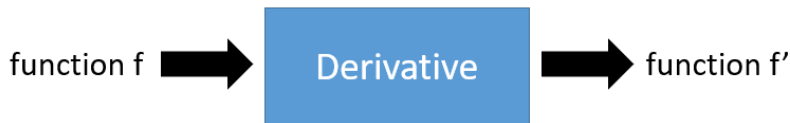
Linearity Check(cont.)-Transpose

Linear : **YES**



Linearity Check(cont.)-Derivative

Linear : **YES**



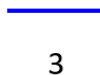
$$f \rightarrow f'$$

$$g \rightarrow g'$$



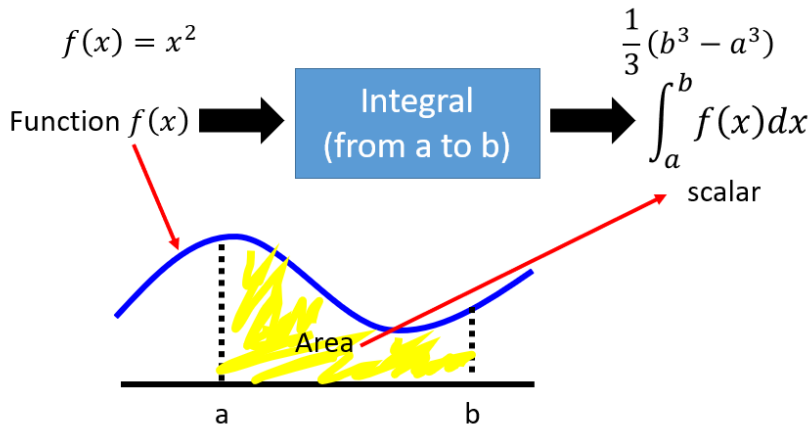
$$kf \rightarrow kf'$$

$$f + g \rightarrow f' + g'$$



Linearity Check(cont.)-Integral

Linear?



Linearity Check(cont.)-Integral

Linear : **YES**

Multiplication

$$f(x) \longrightarrow \int_a^b f(x) dx$$
$$kf(x) \longrightarrow \int_a^b kf(x) dx$$
$$= k \int_a^b f(x) dx$$

Addition

$$f(x) \longrightarrow \int_a^b f(x) dx \quad g(x) \longrightarrow \int_a^b g(x) dx$$
$$f(x) + g(x) \longrightarrow \int_a^b [f(x) + g(x)] dx$$
$$= \int_a^b f(x) dx + \int_a^b g(x) dx$$

Intuitive Guide for Linear Algebra

- An Intuitive Guide to Linear Algebra
- Why Learning Linear Algebra is Important For Data Science And Machine Learning(by Hero Vired)

Introduction of Class(1)

- Instructor

- DoHoon Lee, Ph.D.
- Office :IT Building(102) room 711
- Tel. : 510-2491
- e-mail: dohoon at pusan dot ac dot kr

- Teaching Assistance

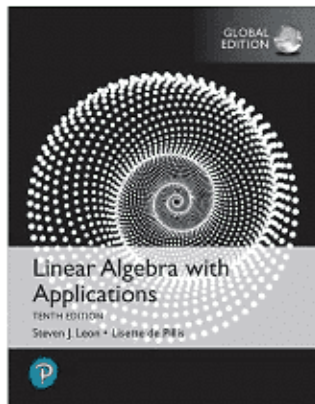
- Meng Socheata(Jia) and Tas Nisa Nur(Nisa)(Master course, VBLab)
- Office : IT Building(Bld 102) room 701(VB Lab)
- Tel. : 510-3469
- e-mail: mengsocheata97 at gmail dot com and nisaanurtas at gmail dot com

- Class Time and Location of Class

- Time : Tue/Thu 10:30-11:45(75 min) class 100
- Classroom : IT Building(Bld 102) 405

Introduction of Class(2)

- Textbook(Requirement)
 - Steve J. Leon, Linear Algebra with Application, 10th edition, 2021.



Grading Policy

- Class Style
 - Lecture
 - Advanced Problems : assignment
- Grading policy
 - Exams: 70 %
 - Assignment : 25 %
 - Etc : 5 %
- Notice
 - Quiz is given in class without notice in advance
 - No grade no exam(mid or final) without reasonable excuse

Class Bulletin Board and Assignment

- Bulletin Board for Class
 - PLATO(PNU smart platform for Learning, Advanced Teaching and Open courseware)
- Assignment policy
 - Only handwritten report(no word processor) but programming assignment
 - The scanned file of the handwritten report should be uploaded to the designed bulletin on PLATO
 - Submit programming or online assignments via PLATO.
- Notice
 - In accordance with the department's internal regulations, this course's grading policy follows the standard policy used for regular Korean classes: grades of A+ - B0 must make up 70% or less of the class(with A+ - A0 not exceeding 40%), and grades of C+ or below must make up 30% or more.

Cheating, plagiarism and illegal reproduction/use

- Attention to cheating(부정행위에 대한 주의) : Forfeit all subject credit and notice to all department(모든 과목 학점 몰수 및 전 학과에 공지)
- Prohibition of academic dishones such as plagiarism(표절 등 학술적 부정행위 금지) : including font, design, and images(글꼴 및 디자인, 이미지 포함됨) : Intellectual property rights(지적 재산권)
- No software illegal use(소프트웨어 불법사용 금지)
- Kim Young-Ran law-related matters(김영란법 관련 유의사항) : Aggressive behavior and credit upgrade(대가성 행위금지, 성적청탁)

Course Schedule (1/2)

- Elementary linear algebra : System of linear equation, elimination and reduction, matrices, determinants.
- Abstract linear algebra : Vector spaces, basis, linear transformations.
- Advanced and applied linear algebra : Orthogonality, inner products, eigenvalue and eigenvector, SVD.

Week	Topic (Leon, <i>Linear Algebra with Applications</i>)
1	Course introduction; Ch.1 Systems of Linear Equations
2	Ch.1 Row Reduction, Echelon Form, Gaussian Elimination
3	Ch.1 Matrix Algebra
4	Ch.2 Determinants
5	Ch.2 Properties of Determinants, Cramer's Rule
6	Ch.3 Vector Spaces, Subspaces
7	Ch.3 Linear Independence, Basis and Dimension
8	Midterm Exam

Course Schedule (2/2)

Week	Topic (Leon, <i>Linear Algebra with Applications</i>)
9	Ch.4 Linear Transformations
10	Ch.4 Matrix Representations, Change of Basis
11	Ch.5 Orthogonality, Inner Product Spaces
12	Ch.5 Gram-Schmidt Process, QR Factorization
13	Ch.5 Least Squares Problems
14	Ch.6 Eigenvalues and Eigenvectors, Diagonalization
15	Ch.6 Singular Value Decomposition (SVD); Review
16	Final Exam

* Schedule may shift slightly depending on class progress and university calendar.

Thanks for your attending My Class, Linear Algebra

Please always ask yourself:

1. What is this ? - Try to understand the essence and nature of the topic at hand.
2. Why is this so ? - Delve into the reasons or principles behind the observed facts or theories.

- From D.H. Lee